Introducing

SOMA™

Parker Brothers Trademark for its Cube Puzzle Game Equipment.

The World's Finest Cube Puzzle Game

Invented by Piet Hein of Denmark

introduction

Welcome to the world of Piet Hein and the most intriguing three dimensional puzzle/game ever devised.

This elegant cube, which is also a *set* of cubes in irregular combinations, will challenge and stimulate your perceptive powers. There are 240 simple ways and 1,105,920 mathematically different ways in which the seven SOMA pieces can be made into the cube.

This booklet poses many challenges. Build the proven structures that can be done. Try to build

those that are not proven...or try to prove that they are impossible.

Explore the competitive possibilities for two or more players.

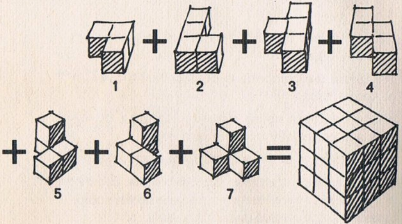
Competitions using different-colored SOMA sets are especially exciting and *new.*

Challenge your friends and your imagination.

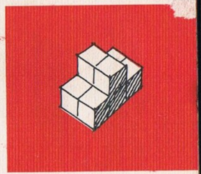
You can also create an almost infinite number of symmetrical or non-symmetrical modern sculptures of your own design.

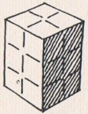
1 challenges for beginners

"Never has anything so easy been so difficult."

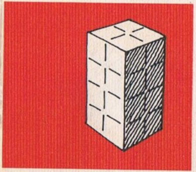


The seven SOMA pieces can be made into the cube in more than one million ways. But try it yourself! There are even more ways in which the cube cannot be made. The structures look so easy. But even with a few pieces, difficult shapes can be made. The shape on the right can be made from two of the seven pieces. Which ones? And how? If you start with this shape it is particularly difficult to build the cube.



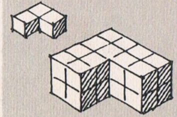


This figure can be made with three of the seven pieces. But which ones? If you start the cube this way, it is believed impossible to complete it.



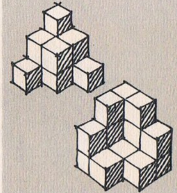
This figure can be made from four of the seven pieces. If you start building the cube this way, you certainly won't be able to finish it because this shape is too tall!

If the smallest of the pieces, No. 1, is removed a similar shape can be made from the remaining six pieces, twice as large in all directions.



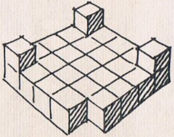
It looks easy! There is a special way of building the cube. First build these two regular figures using respectively three and four of the seven pieces. Which figure *must* use No. 1?

These two figures can then be put together in three different ways to form the cube.

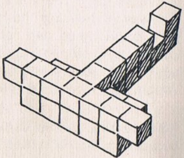


2 some advanced figures

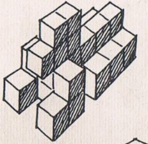
"Buy SOMA sets for your children—but don't let them get hold of them!"



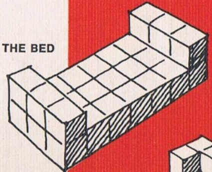
THE CASTLE I



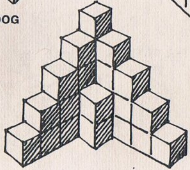
THE AEROPLANE



THE DOG

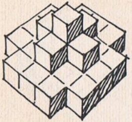


THE BED

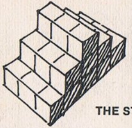


THE CORNER STONE

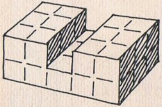
THE TOWER



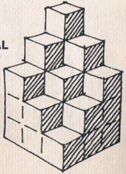
THE PYRAMID



THE STAIRCASE

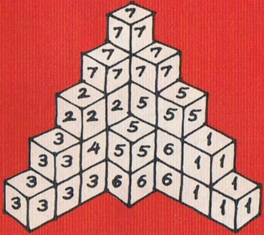


THE CANAL



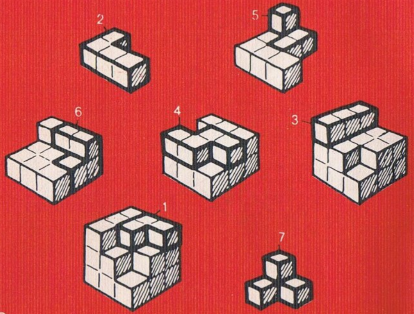
THE CRYSTAL

When you have solved a problem, you can record the solution by filling in the number of the pieces on a drawing:



EXAMPLE: THE CORNER STONE SOLUTION

If you still need help to get started, here is one way of building the cube:



3 now a bit of history about this elegant cube

Condensed from
SCIENTIFIC AMERICAN
and from "Mathematical Games and Puzzles, II"
by Martin Gardner:

From time to time efforts have been made to devise a three-dimensional puzzle game.

None, in my opinion, has been as successful as the SOMA puzzle pieces, invented by Piet Hein, the Danish writer. He conceived of the cube during a lecture on quantum physics. When the lecture touched on a space sliced into cubes, Piet Hein's supple imagination caught a fleeting glimpse of the following curious geometrical theorem:

If you take all the irregular shapes that can be formed by combining no more than four cubes, all the same size and joined at their faces, these shapes can be put together to form a larger cube.

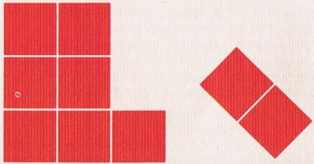
While the lecture continued Piet Hein swiftly convinced himself that there are only seven such pieces. They contain 27 small cubes, so they could form a 3 x 3 x 3 cube. Piet Hein numbered the set of pieces 1 through 7, as shown on page 4.

After working with the pieces for several days, many people find that the shapes become so familiar that they can solve SOMA puzzle problems in their heads. Tests made by European psychologists have shown that ability to solve such problems is roughly correlated with general intelligence, but with peculiar discrepancies at both

ends of the I.Q. curve. Some geniuses are very poor at these puzzles, yet other people of limited ability seem specially gifted with the kind of spatial imagination that these puzzles exercise. Everyone who takes such a test wants to keep playing with the pieces after the test is over.

The number of pleasing structures that can be built with the seven SOMA pieces seem to be unlimited. After I wrote a column about these cubes in *Scientific American*, thousands of readers sent sketches of new figures and many complained that their leisure time had been obliterated since they were bitten by the bug. Teachers purchased SOMA sets for their classes and psychologists added this cube to their psychological tests. Puzzle addicts gave sets to friends in hospitals and as Christmas gifts.

The charm of these puzzles derives in part, I think, from the fact that only seven pieces are used; one is not overwhelmed by complexity.



It is a beautiful freak of nature that the 7 simplest irregular combinations of cubes can form the cube again.
Variety growing out of unity returns to unity.
It is the world's smallest philosophical system.
That is an advantage.

Piet Hein

number of combinations

The seven SOMA pieces can be made into the cube in exactly **1,105,920** different ways, counting as different all solutions which are reflections of each other or that can arise from each other by rotations of the whole cube or of single pieces.

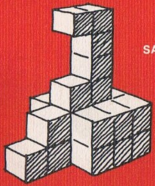
This figure is based on the result of an analysis by Dr. John H. Conway and Dr. M. J. T. Guy, both of Caius College, Cambridge, England, carried out by means of an electronic computer.

The same result was arrived at by N. S. Newhall of the trajectory department at the Jet Propulsion Laboratory of the California Institute of Technology, Pasadena, California, using an IBM 7094 electronic computer which pointed out the solutions in 82 seconds. The result has later been verified by several other scientists.

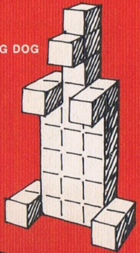
4 figures for experts

"SOMA pieces are sculptures which you yourself sculpt further."

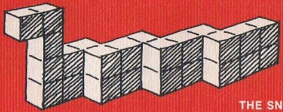
Some of the more difficult classical figures, including several used to illustrate Martin Gardner's article, are reproduced here as drawn by Piet Hein. Solve as many as you can yourself. Even the ones you're sure you *know* can be forgotten when used as the basis for competitions.



THE GALLOWS

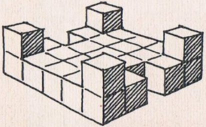


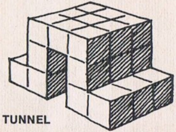
SAM'S SITTING DOG



THE SNAKE

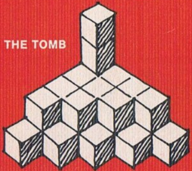
THE CASTLE II



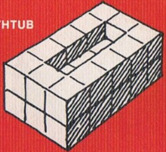


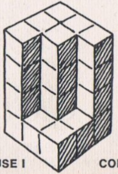
THE TUNNEL

THE TOMB

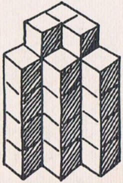


THE BATHTUB

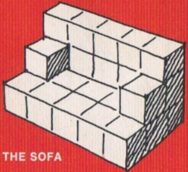




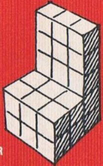
CORNER HOUSE I



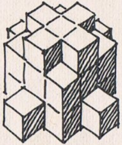
CORNER HOUSE II



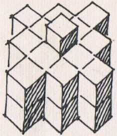
THE SOFA



THE CHAIR

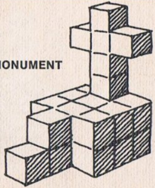


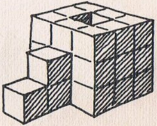
THE GORDIAN KNOT



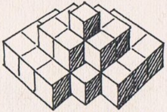
THE MEMORIAL
(Proved impossible by S. Golomb)

THE MONUMENT

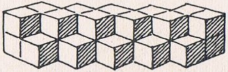




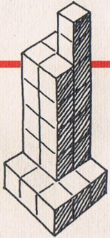
THE WELL



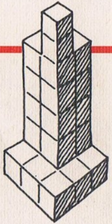
THE STEAMER



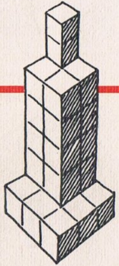
THE FIVE SEATS BENCH



SKYSCRAPER I

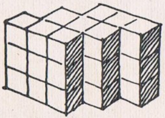


II

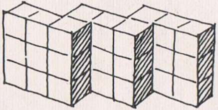


III (Is It Possible?)

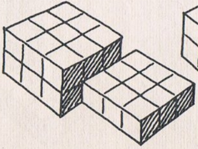
APARTMENT BLOCK I

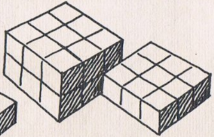


APARTMENT BLOCK II



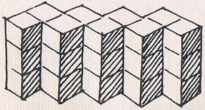
HIGH AND LOW I



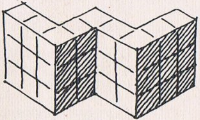


HIGH AND LOW II

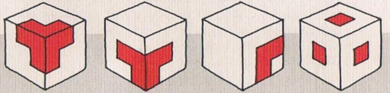
THE ZIG-ZAG-WALL



THE W-WALL (Is It Possible?)



Hundreds of regular figures have been discovered, yet it is still possible to find new ones, now that you are an expert.



Piece No. 7 can, theoretically, have four different positions in the cube—which of them are possible? Such problems lead to additional games.

5 games for two or more players

"Can competitions create new friendships to replace those they destroy?"

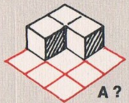
Classical Figure Game

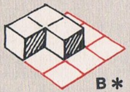
All of the figures illustrated to this point can be used as the basis on which two or more people can compete using one or preferably two SOMA sets. Who can build "The Staircase" or "The Bed" the fastest?

First Piece Placement Game

There are many variations of just one type of game based on building the cube in different ways. For example, with only one set one player may place piece No. 1 in position B (proved possible). The other player must then build the cube "against the clock" using that starting position. With two sets, both players use the same starting piece and position, and race one another. The loser chooses the next starting piece and position. If players have different colored SOMA sets, now available for the first time in the world exclusively from Parker Brothers, Inc., they trade the "challenge" pieces; the color contrasts show clearly that all are attacking the same problem.

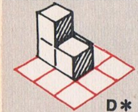
The following Piet Hein drawings show some theoretical cube-building starting positions for each of the seven pieces. Many have been proved possible; those which have an asterisk beside them are possible; those that have a question mark beside them have not been proved possible. It is fun to try to build the cube from the unproven positions on your own; it is particularly exciting to prove one of them in competition.

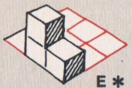

A ?

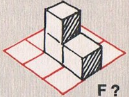

B *

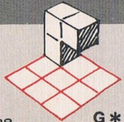

C ?

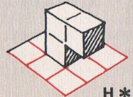
PIECE NO. 1

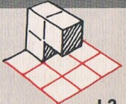

D *

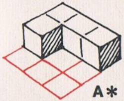

E *

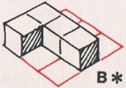

F ?

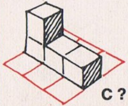

G *


H *


I ?


A *


B *


C ?

PIECE NO. 2

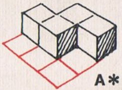

A *

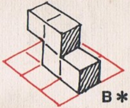

B ?

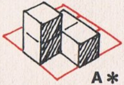

C ?

PIECE NO. 3

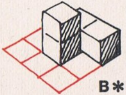
PIECE NO. 4


A *

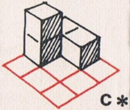

B *



A *



B *



C *

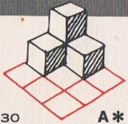
PIECE NO. 5

A ?

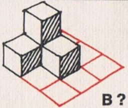
B *

C *

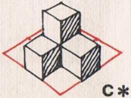
PIECE NO. 6



A *

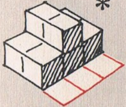


B ?



C *

Now, increase the difficulty of a game by starting the cube with two or more pieces. The following three proven examples are but a few of the many ways to start the cube.



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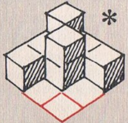

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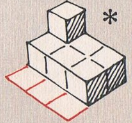

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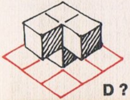


D ?

PIECE NO. 7

Take-Away Game

Create your own challenges by building the cube; then removing all but two, three, or four pieces. Then, see how fast another player can re-build the cube using the removed pieces and the multi-piece starting position you have established. With two or more sets, each player removes the agreed number of pieces working out of sight of one another. Then sets are exchanged.

Non-Standard Sets Game

Another challenging game, also best played with sets of different colors, begins by exchange of any one of your four-cube pieces (Nos. 2, 3, 4, 5, 6, or 7, but not No. 1) for any four-cube piece from another player's set. You will see that the cube can also be built in this dramatic way with both sets non-standard. That is, one might have two No. 2 and no No. 4: the other having the opposite departure from standard. This game has many completely new and delightful strategic

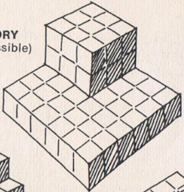
"traps." If any players exchange identical SOMA pieces, trading must of course continue until all competing sets are non-standard.

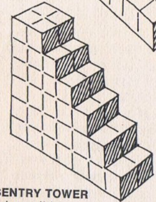
6 two set figures

Using two SOMA sets more than doubles your challenge and enjoyment. Here are Piet Hein's drawings of some proven, and several not proven, two set figures. Try them alone or in double matches. Can you devise other more difficult ones containing fifty-four small cubes?

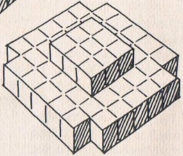
It should now be clear that you can prove a figure possible by building it, but does it follow that those which you have not yet been able to build are not possible? "Not proved possible" and "proved not possible" are two very different things. Jotun's proof (p. 38), illustrates one way of proving a double figure not possible.

1. THE FACTORY
(proved possible)

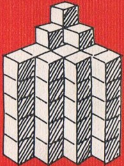




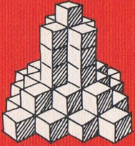
2. THE SENTRY TOWER
(proved possible)



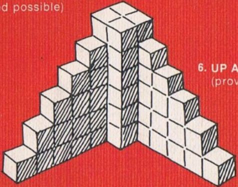
3. THE BRITISH SQUARE
(not proved possible)



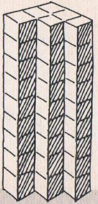
4. THE BIG BUILDING
(proved possible)



5. THE AZTEC TEMPLE
(not proved possible)

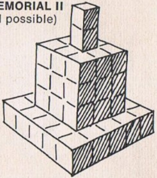


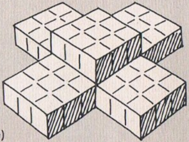
6. UP AND DOWN
(proved possible)



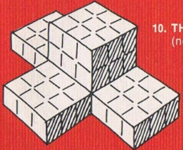
7. THE TRI-TOWER
(not proved possible)

8. THE MEMORIAL II
(proved possible)

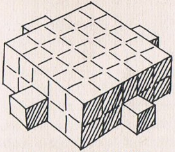




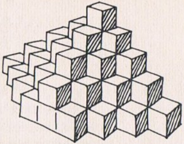
9. THE FIVE SQUARE
(not proved possible)



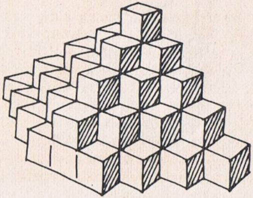
10. THREE WAYS OUT
(not proved possible)



11. THE BLOCKHOUSE
(not proved possible)



12. THE BASALT ROCK
(proved not possible)



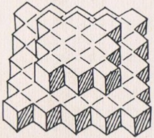
The Basalt Rock (Figure 12) has been proved impossible by Jotun Piet Hein, the twelve-year-old son of inventor Piet Hein.

JOTUN'S PROOF

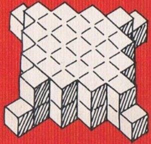
Imagine that the whole structure is checkerboard-patterned in all three dimensions, i.e., that every other cube is white and every other cube

is black—in such a way that the top cube is white. Then all the visible cubes will be white, except the visible center cube on the "ground floor." Counting the white and the black cubes throughout the structures, we find that there are 34 white and 20 black ones, or an excess of 14 white cubes. Can this much excess be brought about among the 14 pieces in two SOMA sets?

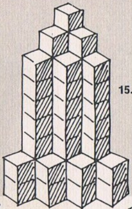
When the SOMA pieces are checkerboard-patterned in all three dimensions, pieces No. 2, 4, 5 and 6 will each have equally as many white and black cubes: two white and two black each, but piece No. 1 will have two of one color and one of the other, and pieces No. 3 and 7 will each have three of one color and one of the other. In other words, a total possible excess of five cubes per set, with only three pieces contributing, respectively, one, two, and two excess cubes of either color. If these excesses all cooperate to make the excess in favor of white as large as possible, then the single SOMA set will have 5 more white cubes than black ones, and two SOMA sets will have, at the utmost, 10 more white cubes than black ones. So a shape that must have 14 more white cubes than black ones cannot be built with two SOMA sets. Can Jotun's proof apply to these figures?



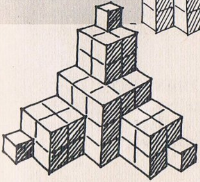
13. ACROPOLIS
(proved possible)



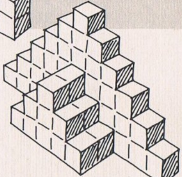
14. SNOWFLAKE
(proved possible)



15. THE GANTRY
(not proved possible)



16. SEPTEMPARTITE
(proved possible)



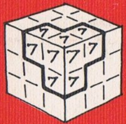
17. THE BUTTRESSED TRIANGLE
(not proved possible)

7 transitions

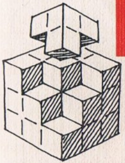
When you are well versed in the building of different shapes, you can begin to explore the fascination of "Transitions"; the magic of how to change from one figure to another. Here are the examples.

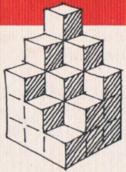
1ST TRANSITION

Build the cube so that piece No. 7 is in the top corner pointing towards you:



turn piece No. 7 around like this:

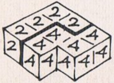


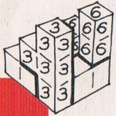


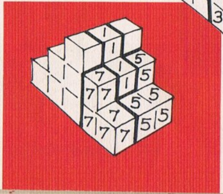
and you have "The Crystal".

2ND TRANSITION

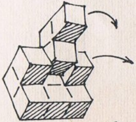
Build the "Staircase" this way:



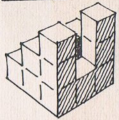


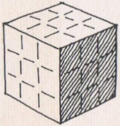


now lift and turn the section made up of pieces No. 1, 5, and 7, letting piece No. 1 swing down as shown:



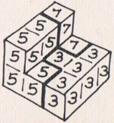
and you have the cube.

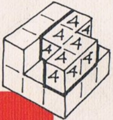


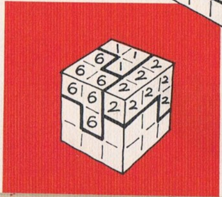


3RD TRANSITION

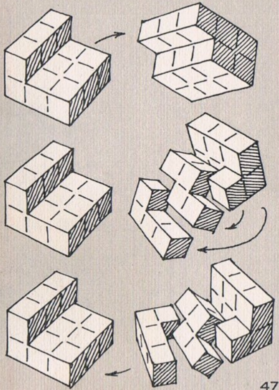
Build the cube this way:



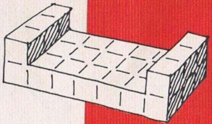




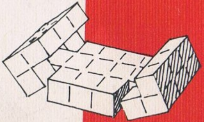
separate it like this:



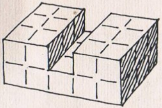
to form "The Bed":



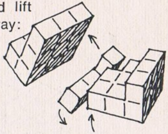
fold the ends in like this:

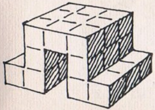


to form "The Canal":



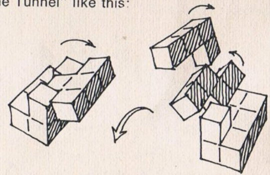
turn it back to "The Bed" and lift the central part of it this way:



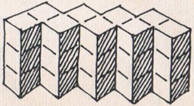


to form "The Tunnel":

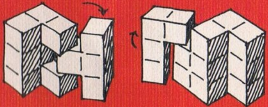
flex "The Tunnel" like this:



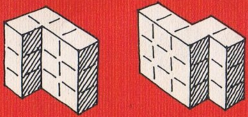
and turn it over onto the front side
to form "The Zig-Zag-Wall":



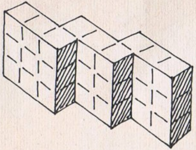
rearrange the Zigs and Zags like this:

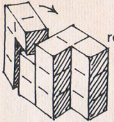


and let the two parts:

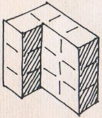


change places so as to form "The Apartment Block II":

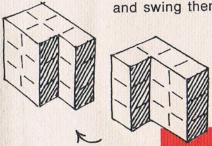


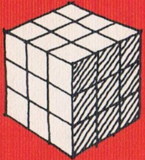


remold the two parts like this:



and swing them around to form the cube again.





any questions?

We will be glad to answer inquiries concerning these rules, if we can. If you would like to contribute puzzles or proofs including impossibility proofs to a periodical for addicts of this game, just write to:

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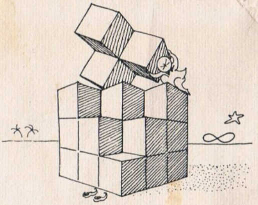
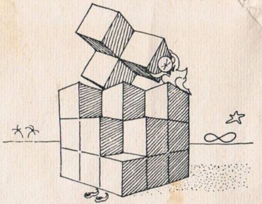


Problems worthy
of attack
prove their worth
by hitting back.

PIET HEIN

Parker Brothers Inc.
SALEM, MASSACHUSETTS / DES MOINES, IOWA



Problems worthy
of attack
prove their worth
by hitting back.

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Invented by Piet Hein of Denmark